Literature Review:

Using Formative Assessment to Overcome Student Misconceptions in Mathematics

Tim Xeriland

Michigan State University

Abstract

Students are plagued with misconceptions, particularly in the field of mathematics. Constructivist theory proposes that student misconceptions be confronted through careful interaction between the student's current thoughts and new ideas to build improved understanding rather than a confrontational approach aimed at replacing faulty ideas with expert knowledge. The field of formative assessment offers a method for this interaction. Attention is paid to what formative assessment is and how terms of the field are defined. The cognitive and motivational theories that underlie formative assessment are explored to provide a coherence of how the separate elements align. Before embracing formative assessment as a method of working with misconceptions the empirical evidence of its effectiveness is looked at critically. Finally, it is examined how formative assessment is put into practice, especially with regards to mathematics.

Key Words: assessment, evaluation, formative, summative, misconceptions, mathematics

Using Formative Assessment to Overcome Student Misconceptions in Mathematics

The longstanding belief has been that as long as there is an is an expert who explains a concept clearly, and as long as there is a student paying attention carefully, that the student will begin to build up the understanding of the expert. Results from educational research tell us that a student may well be paying attention carefully to what is being said, but they are construing it in ways unintended by the expert (Baturo & Nason, 1996). That is, they develop a misconception, which is a student misunderstanding the basic concept of what is taking place (Li & Li, 2008). An important part in remedying this is for teachers to ascertain how the information is being assimilated.

One significant way that teachers can determine how the content is being understood is through assessment and evaluation. However, the traditional approach of evaluation involves students receiving grades for tests, assignments, and papers, and afterwards instruction moves to a new topic. This approach has advantages because it is relatively easy to create, administer, and evaluate tests. The problem is these assessments and evaluations typically fail to offer guidance to students on how their work can be improved. Even if the teacher spends time giving feedback in the form of written comments, as long as a grade is also attached to the evaluation, the comments have been shown to be of no use in helping students improve (Butler, 1988). Dylan Wiliam sums it up by saying, “if you are going to grade or mark a piece of work, you are wasting your time writing careful diagnostic comments” (Wiliam, 1999, p. 8). Possibly because once students receive a final grade, their learned response is to move on to a new topic ignoring all qualitative feedback, hindering their ability to improve on errors.

In contrast, formative assessment requires teachers to access the thought process behind students' answers. Teachers must use this information to give students more opportunity to interact with their misunderstandings. Student misconceptions are commonplace in mathematics education and formative assessment provides teachers with a look into the level of comprehension attained by students when solving problems. This allows teachers to assist students in becoming aware of their misconceptions and improving learning. This elevates formative assessment to the diagnostic level. The problem is that although this approach has been demonstrated to be effective in the classroom it is often superseded by traditional assessment methods (Black & Wiliam, 1998b).

The purpose of this review is to look at recent research to understand the central assertions of misconceptions and the possible benefits gained by utilizing formative assessment strategies. In particular, the review examines the theoretical underpinnings of the constructivist and cognitive framework to this approach.

Misconceptions

The early research on mathematics education viewed student errors as problems that needed to be avoided, and misconceptions as something that needed to be replaced with accurate information (Even & Tirosh, 2008). This stems from a behaviorist view of learning. With this approach the learner is a passive recipient responding to environmental stimuli. The student is viewed as a blank slate (i.e. *tabula rasa*) and behavior is shaped through positive reinforcement or negative reinforcement. For education purposes the underlying assumption is that knowledge can be transferred intact from one person to another and any current knowledge a student has is irrelevant to the learning process (Mamba, 2011).

In contrast to the behaviorist perspective, a constructivist perspective on learning (Smith, diSessa, & Roschelle, 1993) assumes humans generate knowledge and meaning from an interaction between their experiences and their ideas. Here the current knowledge a student has will help determine what the student will learn. With behaviorism it is solely the experience that makes the associations that determine what is learned by the person (Skinner, 1938). With constructivism it is the interaction between cognitive structures and experiences.

An important concept in regards to constructivism is schemas. For the behaviorist, learning simply requires making new associates from the environment, but for the constructivist learning often requires assimilating into existing schemas. Misconceptions are important to this view because they occur when old information in schemas interacts with newly assimilated information (Derry, 1996). These misconceptions can twist and distort information generating errors. Because of the nature of learning, according to this view, misconceptions are considered inevitable and can be used as an important source of information about the learning process (Derry, 1996). This means teachers should regard errors as a clue for uncovering what students already know and how they have constructed such knowledge (Borasi, 1996; Mamba, 2011).

Constructivism emphasizes prior knowledge. Errors are expected in the early stages of learning and are considered to be part of the process. Students recognize the current level of knowledge to be inadequate and learn by “transforming and refining that prior knowledge into more sophisticated forms” (Smith, diSessa, & Roschelle , 1993, p. 123). Many of the assertions of misconception research appear to be inconsistent with constructivism. Misconception research identifies the flaws of students learning. Constructivism "characterizes the process of learning as the gradual recrafting of existing knowledge that, despite many intermediate difficulties, is eventually successful" (Smith, diSessa, & Roschelle , 1993, p. 123). As dictated by constructivist thought, inadequate knowledge is replaced by more sophisticated levels of thinking. Therefore, any misconceptions would be flawed and consequently replaced.

As stated by Smith, diSessa, and Roschelle (1993) two main reasons exist to question the idea of replacement: first, "empirical evidence of the complexity of knowledge learning" and second "considerations of theoretical consistency" (Smith, diSessa, & Roschelle , 1993, p. 125). In the first regard, studies have demonstrated that students can show both correct and flawed thinking processes within the same problem-solving event (Smith, diSessa, & Roschelle , 1993). This shows evidence that cognitive function can support both expert concepts and misconceptions simultaneously. "If concepts are more like complex clusters of related ideas than separable independent units, then replacement looks less plausible as a learning process" (Smith, diSessa, & Roschelle , 1993, p. 125). Secondly, replacement conflicts with the constructivist concept of adapting prior knowledge. This leads to the question of "what prior knowledge is involved in the construction of the expert concepts that replace misconceptions?" (Smith, diSessa, & Roschelle , 1993, p. 125).

To be successful in confronting misconceptions, teachers must provide clear opposition to students' flawed conceptions (Derry, 1996). As students demonstrate flawed thinking, expert ideas must be presented to counteract the misconception. The student is then faced with a choice. They must make a rational choice whether or not to replace their possible misconception with the expert idea presented to them (Derry, 1996). How this takes place is a "complex part of conceptual development and confrontation is an implausible mechanism for changing principles that decide, for example, the relevance of data to theory" (Smith, diSessa, & Roschelle , 1993, p.126).

Replacement of faulty misconceptions is an oversimplification of the complex processes involved in learning complex subject matter. "Literal replacement itself cannot be a central cognitive mechanism, nor does it even seem helpful as a guiding metaphor" (Smith, diSessa, & Roschelle , 1993, p.153). Ideas which initially have appeared to be misconceptions have not in fact been replaced, but seem to have been "refined into more productive forms" and often reappear suggesting that learning processes are in fact much more complex than simple replacement suggests (Smith, diSessa, & Roschelle , 1993, p.153) .

Confrontation appears to undermine students' thinking and confidence in their own ideas rather than engaging them in examination and refinement of their conceptions. (Smith, diSessa, & Roschelle , 1993, p.154) "The goal of instruction should be not to exchange misconceptions for expert concepts but to provide the experiential basis for complex and gradual processes of conceptual change. Cognitive conflict is a state that leads not to the choice of an expert concept over an existing novice conception but to a more complex pattern of system-level changes that collectively engage many related knowledge elements" (Smith, diSessa, & Roschelle , 1993, p.154).

This theory connects nicely with Vygotsky's theory of zone of proximal development (1978). This is the realm between what a learner understands and does not understand and where misconceptions are common place (Barke & Hazari, 2009). To navigate these murky waters Vygotsky's suggests learners need a guide to which they can collaborate. It is only through this experience that a student's understanding will grow. In the next section it will explore a method that allows teacher to consistently work in the zone of proximal development and guide students through their misunderstandings and misconceptions.

Formative Assessment

What is it?

A quick look at the definition of *formative* shows a connection to the idea of forming or shaping something and achieving a beneficial effect. This gets to the central idea of what *formative assessment* is intended to do, which is "shape and improve the students' competence" (Sadler, 1989, p. 120). It is common in the literature on formative assessment to see two general ideas on how this molding is done. The first idea is using assessment not to grade, but to give feedback to the student that gives specific guidance that the student is required to utilize (Black & Wiliam, 1998b). The second notion is that the assessment is used to "gain information that can help the teacher plan effective instruction" (Ginsburg, 2009, p. 110). Because of the varying ideas of what constitutes formative assessment a good general definition, that is common in the literature, (Chappuis & Stiggins, 2002; Waterman, 2010) is "assessment for learning" (Dunn & Mulvenon, 2009, p. 3).

The etymology of *summative* is associated with summing up or a conclusion. This fits well with the concept of *summative assessment* which "is geared towards reporting at the end of a study" (Sadler, 1989, p. 120). Taras (2009) states summative assessment is "when the assessment stops at the judgment" (p. 58). This kind of assessment is often considered the polar opposite of formative assessment. It is convenient to think of summative assessment with regards to a student receiving a grade, but the term summative is not used consistently in the literature (Black & Wiliam, 1998a). A good general definition for summative assessment is "assessment of learning" (Dunn & Mulvenon, 2009, p. 3).

A careful examination of the formative assessment literature demonstrates a vagueness of operational definitions. The widespread confusion stems from a lack of agreed upon definitions. For example, Black and William (1998a) defined assessment as "all those activities undertaken by teachers, and/or by their students, which provide information to be used as feedback to modify the teaching and learning activities in which they are engaged" (p. 17) . These activities could include reading and writing exercises, lecture checks, group work, board work, class debates, etc. The Council of Chief State School Officers (CCSSO) defines formative assessment as "a process used during instruction to provide feedback for adjustment of ongoing teaching and learning for the purposes of improving student achievement related to instructional objectives" (Heritage, 2010, p. 9). Taras (2009) even pointed out that Black and Wiliam have two different and distinct definitions of formative assessment in the same paper.

Moreover, much of the assessment literature is devoted to making a clear separation between summative and formative assessment. Summative assessment, however, can be used with formative intentions (Bell & Cowie, 2000). For Black and William (1998b) an assessment is only formative if it is used to provide feedback to the teacher and/or student. For them "feedback to any pupil should be about the particular qualities of his or her work, with advice on what he or she can do to improve, and should avoid comparisons with other pupils" (Black & Wiliam, 1998b, p. 142). Other researchers define feedback as information about the gap between actual and predetermined levels (Ramaprasad, 1983; Sadler, 1989). Usually it will be mentioned that the feedback must be used to modify the teaching and learning process for formative assessment to have taken place. Technically speaking, however, an assessment that assigns only a grade (i.e., “A” or “B”) does provide feedback and could be used in the future to adapt the teaching and learning process. This would be classified as formative assessment under many researchers’ definitions (Dunn & Mulvenon, 2009).

Even more confusing Taras (2009) believes breaking formative and summative assessment into two divisions is a false dichotomy. She pointed out that for formative assessment to take place a judgment of the quality of work must have taken place and this judgment is a summative assessment. Sadler (1989) also believed that the initial assessment must be a summative assessment and by not naming the initial step of summative assessment, the whole process of formative assessment seems to have become confused by Black and Wiliam (Taras, 2009). This well meaning attempt to meld summative assessment to formative assessment seems to have caused even more confusion in a field already fraught with disorder.

Formative assessment as topic of research interest can be followed back to Scriven’s (1967) description of *formative evaluation* in education (Allal & Lopez, 2005). Some researchers have traced the complications of the field back to when Bloom (1969) transferred the term formative from evaluation to assessment (Dunn & Mulvenon, 2009). It has been seen that an assessment can be designed for a certain purpose, but teachers can use the data gathered in any method they see fit (Wininger, 2005). In other words, regardless of the purpose of the assessment, the data can be interpreted either formatively or summatively. Dunn and Mulvenon (2009) argue that “definitions of formative and summative assessments that include how the data is used leads to issues in the literature due to the possibility of evaluating or using either type of assessment data formatively or summatively” (p. 3). For them it adds clarity to the field to separate assessment from evaluation. These researchers define the terms formative evaluation and summative evaluation in regards to the use of assessment data and separate the issue of assessment instruments from assessment use (Dunn & Mulvenon, 2009).

Theory & Research

Cognitive and motivational theories lie beneath formative assessment. Understanding these theories is crucial to explain how formative assessment works and why. This understanding is also important because the theoretical basis for formative assessment ties together separate elements of effective practice and helps to clarify them and see how they fit together.

Cognitive. From the constructivist perspective learners make sense of new information when it is incorporated in mental constructs or schemas (Smith, diSessa, & Roschelle, 1993). Often a teacher's job is to help students' organize how well their schemas are connected (Donovan & Bransford, 2005). This is done by having the student interact with where they currently are and where they need to be. This interaction can come from teachers, peers, or even carefully diagnostic feedback (Reveles, Kelly, & Durán, 2007). This interaction works best if it is close to the boundary of what a student knows and does not know, called the zone of proximal development (ZPD) (Vygotsky, 1978). For Vygotsky (1978) learning was a social process that required interacting with others.

Vygotsky's theory distinguishes two levels of development. The first is the current level of the individual. The second is the level of potential development. This potential development is the level that the student is capable of reaching with the assistance of a teacher or peers (Heritage, 2010). To guide in this process it is important to build scaffolding for the student

(Belland et al., 2008). For example a teacher may ask "leading or probing questions to elaborate the knowledge the learner already possesses, or providing feedback that assists the learner to take steps to move forward through the ZPD" (Heritage, 2010, p.8). As the learner becomes more competent in the subject area the scaffolding is gradually reduced until the learner is able to function independently (Heritage, 2010; Tharp & Gallimore, 1988).

The connection to formative assessment is clear. Formative assessment enables teachers and students to consistently work in the zone of proximal development (Heritage, 2010). By its nature formative assessment involves an interaction or a dialogue between teacher and student and it is this interaction that helps to build what Vygotsky (1978) calls "maturing factors" (p. 86). By carefully utilizing formative assessment teacher can determine what is within the reach of their students and provide useful experience that support learning (Heritage, 2010). In short, formative assessment helps students assimilate new information into their schemas and thereby follows the theory laid out by the constructivist approach.

Feedback. For Sadler (1989) feedback is the crucial factor to aid learning. It is usually defined by giving information about how something is being done. Sadler (1989) instead prefers a systems perspective of feedback and uses Ramaprasad (1983) definition "information about the gap between the actual level and the reference level of a system parameter that is used to alter the gap in some way" (p. 4). With this approach feedback loops are important and formative assessment is the key to closing the gap.

To create these feedback loops requires students to practice in a supportive environment that usually includes a teacher who understands the skills that are needed for improvement and how to correct a poor performance. For Sadler's model the feedback is used both by teachers to make programmatic decisions and also by students to monitor their strengths and weaknesses. The key is that it is only feedback "*when it is used to alter the gap*" and is useless if the "information is simply recorded" (Sadler, 1989, p. 121).

Effective feedback focuses on the task and provides the student with suggestions, hints, or cues; feedback in the form of praise is problematic (Kluger & DeNisi, 1996). For the feedback to be successful it must be prompt and accurate. The closer the feedback comes to the assessment the more effective it is for student achievement (Waterman, 2010). Wiliam and Leahy (2007) advise three *time scales cycles* for feedback: short, medium, and long. Short is defined as between five seconds and one hour. Medium is between one day and two weeks and long is between four weeks and one year.

Short time scale feedback is usually done to help a teacher decided whether to proceed to a new topic or explain the content again. Medium time scale feedback is better used to determine if an entire class needs to be retaught or to take aside those students that have not learned to reteach them in a special session. Long time scale feedback is less common and is used to determine if students are needed to be regrouped for reteaching learning objectives (Wiliam & Leahy, 2007; Waterman, 2010).

Further evidence points to the role of effective feedback in the learning process. In a review of 196 studies describing nearly 7,000 effects it was reported that feedback had an average effect size of 0.79 (Hattie & Timperley, 2007; Heritage, 2010). If realized, this size effect would be one of the largest in educational interventions and would have a profound impact on a student's success. Sadler, however, was concerned with the common but puzzling observation that even when teachers provide feedback there may be no improvement. This is explored in the next section on motivation.

Motivation. Attribution theory is a good place to start when looking at what motivates students academically. It focuses on how people attribute the causes of events and how these judgments influence internal perceptions. It is tied with the concept of self-efficacy, which are students' beliefs in their own competence. The students' self-efficacy will play a role in how they will interpret success or failure.

In mathematics the self-efficacy of student's can be defined as their internal perceptions of the potential for their success (Wolters & Rosenthal, 2000). It has been shown that students with higher levels of self-efficacy set higher goals, apply more effort, and persist longer on difficult tasks (Tanner & Jones, 2003). Students with this high self-efficacy do not believe that mathematical ability is fixed and they attribute their success or failure to internal factors (Black, 1998). Black (1998) worries that when a student repeatedly gets low grades it may cause "a shared belief between them and their teacher that they are just not clever enough" (p.43). Thus, low self-efficacy can be the unintended result of summative assessment (Tanner & Jones, 2003). This can create a vicious circle where a student fails, which lowers self-efficacy that results in more failure.

A study by Butler (1988) demonstrated that giving grades does boost students' interest only when the grades are high. As the grades dropped interest plummeted. In another study by Butler (1987) she broke 200 students into four groups and individuals of each group were given one of four kinds of feedback on a lesson: comments, grades, praise, and no feedback at all. On the second lesson only those who received comments showed improvement over the first lesson. At this time students were also given a questionnaire to see if they attributed their success or failure to ego-involvement or task involvement. Those who were given comments had high levels of task-involvement but their ego-involvement was the same as those that received no feedback at all (Wiliam, 1999). While the two groups that received praise or grades both had high ego-involvement and low task involvement. In addition, they demonstrated no improvement in performance. Is having high ego-involvement a good thing if it does not improve performance?

In a review of 131 studies on feedback it was found that, on average, feedback did improve performance (Wiliam, 1999). A closer look shows a significant difference between studies. A whopping 40% of the studies showed giving feedback actually had a negative affect on performance (Wiliam, 1999). On further investigation it was found that those studies where feedback hurt performance the feedback was focused on self-esteem or self-image (as in the case of grades or praise) (Wiliam, 1999). This suggests the importance of the kind of feedback used. In particular formative feedback is recommended (Black & Wiliam, 1998b). Feedback is considered formative "only if the information fed back to the learner is used by the learner in improving performance" (Wiliam, 1999, p.8).

Feedback promises to be a powerful performance improver. Looking more closely at research, however, demonstrates that certain kinds of feedback are more promising than others. In a similar manner formative assessment had been touted to be highly effective. In the next section it will look at the effectiveness of formative assessment.

A Critical Review

For many it is a given that formative assessment improves student performance. After all, not only do Black and Wiliam (1998a) state that the evidence is conclusive, but that gains under its use are "amongst the largest ever reported" (p. 61). Black and Wiliam's conclusion is drawn after reading 700 journal articles on assessment and narrowing the list down to 250 as being relevant. From this they whittle it down to, what are deemed, eight top-notched research studies that give them their evidence that formative assessment does work. However, some researchers have found issues with all eight studies (Dunn & Mulvenon, 2009).

The biggest problem with the studies is issues with generalziability. Black and Wiliam's conclusion most strongly relies on the meta-analysis of Fuch and Fuch (1986). While this had 3,835 participants, from the various studies, 83 percent were handicapped. This is because the review was specifically focused on special education. Although there was an average effect size of 0.63 for the non handicapped participants it seems inappropriate to generalize this to all students given the slant of the sample (Dunn & Mulvenon, 2009).

Several of the studies used by Black and Wiliams do not account for teacher effects. For example, one study had a massive sample of 7,000 students viewed over eighteen years (Whiting et al., 1995). Only one teacher, however, was used in the study. While the teacher did use formative assessment and was directly compared to a teacher who did not use formative assessment it is difficult to ignore the potential confounding variables.

Dunn and Mulvenon (2009) reviewed nine more recent articles on formative assessment. While the articles do lend more evidence to formative assessment they also suffer from methodological issues. For example, one promising study gave impressive results, but is difficult to generalize due to the sample size of four. In addition, while Black and Wiliam (1998) reported effect sizes of 0.70 the effect sizes of more recent studies tend to be smaller (Buchanan, 2000).

While it is a popular notion in education that formative assessment has already been proven to work, clearly more research needs to be done. The topic has both great potential and vulnerability due to the lack of strong empirical evidence. There are many questions that can be explored further. For example, are there certain subjects that benefit more than others? Is there a best way to use formative assessment for certain disciplines? Since misconceptions are so prevalent in mathematics (Ginsburg, 2009) and formative assessment allows for the interaction and opportunity for a student to wrestle with inconsistencies, which theory suggests is an ideal way to approach them, how should formative assessment be applied to the study of mathematics? This question will be explored further in the next section.

Formative Assessment - Mathematics

In mathematics it is important for the teacher to understand how students solve problems (Wiliam, 1999). This knowledge lends insight into the thought process of students and makes it easier for teachers to overcome their students' conceptual difficulties. In this case, formative assessment is diagnostic. It has been shown that, in particular, formative assessment can give information about a student's performance, thinking, and learning potential (Ginsburg, 2009).

When researcher Herbert Ginsburg (2009) started with the question, "How can the new psychology of mathematical thinking be used to improve mathematics education?" (p. 109). His conclusion was, "to inform, indeed transform, the process of 'formative assessment'" (Ginsburg, 2009, p. 109). For Ginsburg (2009) using formative assessment in mathematics starts with an understanding of the three methods that it can be employed by: observation, clinical interview, and test.

The observation method has a long history in psychology (Piaget, 1952). Piaget actively observed his children to develop his stages of development theory. Ginsburg (2009) states, “Much can be learned from keen observation. We see that a child playing with two blocks is exploring ideas of symmetry and pattern” (p. 112). It is rare that observation is pure and typically involves some interaction with the subjects being observed (Ginsburg, 2009). Also, it is not as simply as it may seem. Generally it involves a great deal of thinking about what to be on the lookout for. This type of assessment would be very difficult to do in a large classroom setting.

Because it is not often convenient to wait for students to do something important to observe, the clinical interview is another approach. In this case, the interviewer develops a task and asks the subject to complete it. This is useful because the interviewer can stop the subject and ask questions along the way. For example, “what makes you decide to start at that point?” or “how did you do that?” Typically the interviewer would start with an assumption and give a task while asking questions in an attempt to verify the hypothesis. This can be difficult because it requires the interviewer to make interpretations and act quickly on those interpretations (Ginsburg, 2009). Because this is a focused method that still offers a teacher a lot of flexibility it has large benefits. For Ginsburg (2009) this is the most powerful formative assessment technique and offers “deep insight into thinking” (p. 115). Again, this would be difficult to employ in a classroom setting, but useful for one-on-one situations.

While the other two approaches are used far less in education the most common formative assessment method is the use of tests or tasks. These have the benefit of getting right to the important questions a teacher wants to ask. Ginsburg (2009) believes much can be learned from tests and it is not just checking for right and wrong answers, but "strategies of solution" (p. 113). Ginsburg (2009) writes that the teacher "may observe what [a student] writes on paper to solve a problem, thus obtaining information about the strategy" (p. 113). If observing the students' written work is not possible, it is still possible to discover the thought processes behind the errors using a carefully crafted multiple choice test, which gives known misconceptions as answer choices (Ginsburg, 2009).

At the same time Ginsburg points out the drawbacks of tests. He tells how Piaget (1952) reported that working as an administrator of standardized intelligence test he found responses to be of less interest than the cognitive process that produced them, but those could not be inspected because of the technique of standardized test administration (Ginsburg, 2009). The idea is that using formative assessment in testing is useful not for the answers, but instead the thought process behind the answers.

Great value can be placed upon the use of mathematics to further understand the complex nature of cognitive development, it "is essential for teaching and for assessment as well" (Ginsburg, 2009, p. 115). Ginsburg (2009) identifies three vital aspects of formative assessment: normative information, cognitive trajectory, and trajectory of mathematical ideas.

Normative information is concerned with performance. In this case it requires a teacher have insight into what is a typical performance level for students of a given age. When teachers are cognizant of how their students should be performing it allows them to see when remediation is needed.

In addition to normative information cognitive trajectory must be utilized. As children learn simple mathematical principles, the overall progression begins with rote memory. Children start with the memorization of the first ten numbers or so. They progresses to the detection of counting patterns in the numbers above 20. Then students realize the decade numbers are multiples of 10 and that means the unit numbers can be added to the decade numbers to form our counting system; with this comes understanding of the underlying base ten composition (Ginsburg, 2009).

Lastly it is equally important that teachers understand the trajectory of mathematical ideas. For example, suppose that a first-grade child is struggling with counting past 40. He is able to count to 39 without much trouble but is unable to produce 40. When given that number he is able to continue 41-49 but is unable to produce 50. What seems to be a minor, isolated issue with counting can in fact have far reaching significance with greater ideas. “It is part of a larger learning trajectory involving the development of base ten concepts” (Ginsburg, 2009, p. 116). As noted by Ginsburg (2009), “formative assessment must rest on a foundation of mathematical understanding and can benefit from considering three kinds of developmental trajectory – performance, cognitive and mathematical” (p. 117).

As children assimilate new information into what they already know, misconceptions can result. Often the child takes a principle that might be useful in certain contexts and generalizes it in an inappropriate manner. As seen with subtracting a smaller number from a larger: 12 – 9 = 17 (Ginsburg, 2009). This is a misconception which must be corrected. The teacher may ask the child to use blocks to provide a real object to face the conflict. Once the child is able to see the correct result using the blocks, he must then understand the same is true with written mathematics. "The child needs to be convinced written mathematics should make sense" (Ginsburg, 2009, p. 125).

Conclusions

While formative assessment has been around for numerous years, in many ways, it is still in its infancy. That is, the field is still grappling with presenting a unified definition of basic terms. More attention to the theoretical underpinnings could bring coherence to the ideas and concepts and how they can work together. While students of mathematics will continue to have misconceptions it remains to be seen if formative assessment will bring improvement. In general, the studies show great promise, but more work is needed. In particular the area of feedback holds great hope. As the research ties itself closer to theory it can work to find the best manner in which feedback is given. By using the constructivist framework, and bringing in cognitive and motivational theory, researchers could develop best practices around formative assessment, and achieve not only outstanding performance gains for students, but increased motivation and interest to learn as well.

References

Allal, L., & Lopez, L. M. (2005). Formative assessment of learning: A review of publications in French. In OECD (Ed.), *Formative assessment: Improving learning in secondary classrooms* (pp. 241-264). France: OECD Publishing.

Barke, H. D., & Hazari A. (2009). Students' Misconceptions and How to Overcome Them Misconceptions in Chemistry - New York, NY: Springer.

Baturo, A., & Nason, R. (1996). Student teachers' subject matter knowledge within the domain of area measurement. *Educational Studies in Mathematics*, 31, 235-268.

Belland, Brian., Glazewski, Krista D., and Richardson, Jennifer C. (2008). A scaffolding framework to support the construction of evidence-based arguments among middle school students. *Education Tech Research Dev*., 56, 401-422.

Bell, B., & Cowie, B. (2000). The characteristics of formative assessment in science education. *Science Education*, 85, 536-553

Black, P. (1998). Formative assessment: raising standards inside the classroom. *School Science Review,* 80(291) 39-46.

Black, P., & Wiliam. D. (1998a). Assessment and classroom learning. *Assessment in Education*, 5(1), 7-74.

Black, P., & Wiliam, D. (1998b). Inside the black box. *Phi Delta Kappan*, 80(2), 139-148.

Bloom, B. S. (1969). *Some theoretical issues relating to education evaluation.* In: TylerRW, (ed.) Educational Evaluation: New Role, New Means. Chicago, IL: University of Chicago Press.

Borasi, R. (1996). *Reconceiving Mathematics Instruction: A Focus on Errors*. Norwood, New Jersey: Ablex Publishing Cooperation.

Buchanan, T. (2000). The efficacy of a world-wide web mediated formative assessment. *Journal of Computer Assisted Learning*, 16(3), 193-200.

Butler, R. (1988). Enhancing and undermining intrinsic motivation: The effects of task-involving and ego-involving evaluation on interest and performance. *British Journal of Educational Psychology*, 58, 1-14.

Butler, R. (1987). Task-involving and ego-involving properties of evaluation: effects of different feedback conditions on motivational perceptions, interest and performance. *Journal of Educational Psychology,* 79, 474-482.

Chappuis, S., & Stiggins, R. J. (2002). Classroom assessment for learning. *Educational Leadership*, 60(1), 40-43.

Derry, Sharon J. (1996). Cognitive Schema Theory in the Constructivist Debate. Educational Psychologist. 31(3/4), 163-174.

Donovan, M. S., & Bransford, J. D. (2005). Introduction. In M. S. Donovan & J. D. Brandsford (Eds.), *How students learn: History, mathematics, and science in the classroom* (pp.1 28).Washington, DC: National Academic Press.

Dunn, K., & Mulvenon, S. (2009). A critical review of research on formative assessment: The limited scientific evidence of the impact of formative assessment in education. *Practical Assessment, Research & Evaluation*, 14(7), 1-11.

Even, R., & Tirosh, D. (2008). Subject-matter knowledge and knowledge about students as sources of teacher presentations of the subject matter. *Educational Studies in Mathematics*, 29(1), 1-20.

Fuch, L. S., & Fuch, D. (1986). Effects of systematic formative evaluation: A meta-analysis. *Exceptional Children*, 53, 199-208.

Kluger, A. N., & DeNisi, A. (1996). The effects of feedback interventions on performance: A historical review, a meta-analysis, and a preliminary feedback intervention theory. *Psychological Bulletin*, 119, 254-284.

Hattie, J., & Timperely, H. (2007). The power of feedback. *Review of Educational Research*, 77, 81-112.

Heritage, (2010). *Formative assessment and next-generation assessment systems: Are we losing an opportunity?* Washington, DC: Council of Chief State School Officers. Retrieved July 30, 2011 from http://www.ccsso.org/Resources/Publications/Formative_Assessment_and_Next Generation_Assessment_Systems.html

Ginsburg, H. (2009). The challenge of formative assessment in mathematics education: Children's minds, teachers' minds. *Human Development* (0018716X), 52(2), 109-128.

Li, X., & Li, Y. (2008). Research on students' misconceptions to improve teaching and learning in school mathematics and science. *School Science and Mathematics*, 108(1), 4–7.

Mamba, F. T. (2011). An investigation into students' misconceptions in linear equations in public secondary schools of malawi: The case of the south eastern education division. Hiroshima, Japan: Hiroshima University.

Piaget, J. (1952). *The origins of intelligence in children*. New York: International Universities Press.

Ramaprasad, A. (1983). On the definition of feedback. *Behavioral Science*, 28, 4-13

Reveles, J. M., Kelly, G. J., & Durán, R. P. (2007). A sociocultural perspective on mediated activity in third grade science. *Cultural Studies of Science Education*, 1, 467-495.

Sadler, D. (1989). Formative assessment and the design of instructional systems. *Instructional Science*, 18, 119-144.

Scriven, M. (1967). The methodology of evaluation. In R. W. Tyler, R. M. Gagne, and M. Scriven (Eds.), *Perspectives of curriculum evaluation*, Volume I (pp. 39-83). Chicago, IL: Rand McNally.

Skinner, B.F. (1938). *The behavior of organisms*. New York: Appleton-Century-Crofts.

Smith, J., Disessa, A., & Roschelle, J. (1993). Misconceptions reconceived: A constructivist analysis of knowledge in transition. *The Journal of the Learning Sciences*, 3(2), 115-163.

Tanner, H. & Jones, S. (2003). Self-efficacy in mathematics and students' use of self-regulated learning strategies during assessment events. Proceedings from 27th International Group for the Psychology of Mathematics Education. PME: HI.

Taras, M. (2009). Summative assessment: the missing link for formative assessment. *Journal of Further and Higher Education*, 33(1), 57-69.

Tharp, R. G., & Gallimore, R. (1988). *Rousing minds to life: Teaching, learning, and schooling in social context*. Cambridge, MA: Cambridge University Press.

Vygotsky, L. S. (1978). *Mind and society: The development of higher mental processes.* Cambridge, MA: Harvard University Press.

Waterman, S. S. (2010). *Assessing middle and high school mathematics and science: Differentiating formative assessment*. Larchmont, NY: Eye On Education.

Whiting, B., Van Burgh, J. W., & Render, G F. (1995). *Master learning in the classroom.* Paper presented at the Annual Meeting of the American Educational Research Association, San Francisco, CA.

Wiliam, D. (1999). Formative assessment in mathematics part 2: Feedback. *Equals: Mathematics and Special Educational Needs* 5(3) 8-11.

Wiliam, D., & Leahy, S. (2007). A theoretical foundation for formative assessment. In J. McMillan (Ed.), *Formative classroom assessment: Theory into practice*. NewYork, NY: Teachers College Press.

Wininger, R. S. (2005). Using your test to teach: Formative summative assessment. *Teaching Psychology*, 32(2), 164-166.

Wolters, C. A., & Rosenthal, H. (2000). The relation between students' motivational beliefs and their use of motivational regulation strategies'. *International Journal of Educational Research*, 33, 801-820.